Scientific Computing

MATH6183001

**TAYLOR SERIES**

Sequence: ordered set of numbers {T1, T2, T3, ….., Tn}.

Series: sum of a sequence up until n terms: = T1 + T2 + T3 + …+Tn.

McLaurin Series (at x = 0)

Ex:

Find the McLaurin series about x = 0 and write it in sigma notation.

.

Taylor Series (about x = a)

Rn = remainder =

Example:

Compute a Taylor Series expansion for f(x) = 5x2 + 3x + 5 around a = 0 and a = 1. Show that f(x) and the Taylor series are identical.

Ex: Approximate using Taylor series (3 non-zero terms), 4 d.p

In numerical analysis there are 2 type of errors:

1. round-off error

2. truncation error

Ex:

4 terms, e2 = 6.3333

5 terms, e2 = 7.0000

6 terms, e2 = 7.2667

….etc → there is truncation error.

**Estimating Truncation Errors**

Taylor series n terms

= truncation error

; a < z < x.

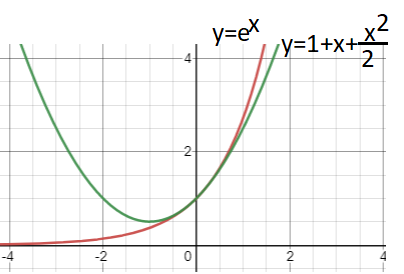
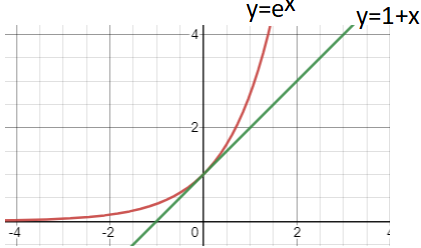
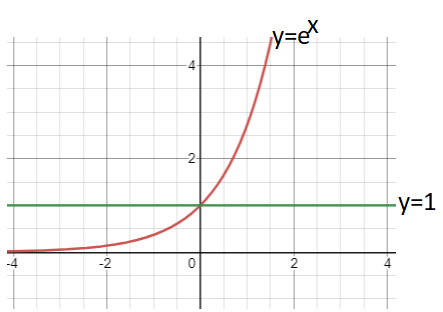
; M = max of

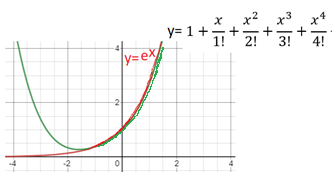
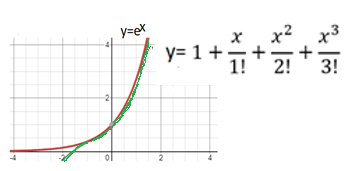
Example:

Estimate the remainder bound for the approximation using Taylor series for e2 using n = 9 around a = 0.

**Visualization**

at x=0.





Ex:

1. Given f(x) = ln (x).

a. Estimate ln(1.1) by 4th order of Taylor series at x = 1.

b. Find the maximum error.

c. Find the actual error.

d. Compare b and c.

2. Given f(x) = .

a. Estimate by 2nd order of Taylor series at x = 1.

b. Find the maximum error

c. Find the actual error.

d. Compare b and c.

3. Given f(x) =

a. Estimate by 3rd order of Taylor series at x = 0.

b. Find the maximum error

c. Find the actual error

d. Compare b and c.